Math 2050, HW 3

Q1. Show that

(a) $\lim\limits_{x \to 1} \dfrac{x}{1+x} = \dfrac{1}{2}$;

(b) $\lim\limits_{x \to 1} x^2 + 3x = 4$;

(c) $\lim_{x \to 0+} \sqrt{x}\sin(x^{-1}) = 0$.

Q2. Show that the following limit does not exists.

(a) $\lim_{x \to 1}(x-1)^{-2}, x > 0$;

(b) $\lim_{x \to +\infty} \sin x$;

Q3. Define

$$f(x) = \begin{cases} x, & \text{if } x \in \mathbb{Q}; \\ 0, & \text{otherwise.} \end{cases}$$

Show that f has limit at $x = 0$ and does not have a limit at $c \in \mathbb{R}$ for all $c \neq 0$.

Q4. Let $: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. If f has a limit L at $x = 0$.

(a) Show that $L = 0$;

(b) Show that f has a limit at every $c \in \mathbb{R}$.